APPENDIX C
Mann-Whitney U-Test*

The random variable to be analyzed shall be the concentrations of the individual contaminants of concern in each individual monitoring well. The statistic to be evaluated is the Mann-Whitney "U". The test shall be a Mann-Whitney U-test with the size of the test equal to 0.1. The hypotheses (H) to be tested are:

H_0: $\hat{\theta}_1 \leq \hat{\theta}_2$ (null hypothesis)
H_1: $\hat{\theta}_1 > \hat{\theta}_2$ (alternate hypothesis)

where $\hat{\theta}_2$ represents the stochastic size of the population of each individual contaminant during the most recent 12 month period of sampling and $\hat{\theta}_1$ represents the stochastic size of the population of each individual contaminant during the previous 12 month period. The test is applied to each contaminant in each individual monitoring well. In other words, if benzene and trichloroethene are the contaminants of concern, and there are four monitoring wells involved in the sampling program, then a total of eight Mann-Whitney tests are to be performed (benzene in each of the four monitoring wells and trichloroethene in each of the four monitoring wells).

The U statistic shall be evaluated as follows:

1. The test is applied to eight consecutive quarters of analytical data for each individual contaminant in each individual monitoring well.

2. For each quarter of data, annotate the concentration of the specific contaminant in the specific monitoring well with either a "b" for the most recent four quarters or an "a" for the four quarters from the previous 12 month period.

3. Vertically arrange the eight contaminant concentrations, with notations, in order of increasing value: the lowest value on the top, and the greatest value on the bottom.

4. For each individual "a" concentration, count the number of "b" concentrations that occur below that "a" concentration in the column.

5. Add the four values (zero or some positive number) obtained for Step 4 to calculate the "U" value.

6. All values of non-detectable (ND) or values detected below the limits of quantitation are to be ranked as "zero." It is required that appropriate detection levels/quantitation limits be achieved.

7. If two or more concentrations are identical, then two vertical columns are necessary. In the first column, rank tying "b" concentrations first, and in the second

column rank tying "a" concentrations first. Calculate an interim "U" for each column ("Ua" and "Ub"). The average of these interim values is the actual "U". This is shown in Example 2, below.

The hypotheses shall be tested as follows:

1. If "U" is three or less, the null hypothesis is rejected, and it is concluded, with at least 90 percent confidence, that the concentration for the individual contaminant has decreased with time at the specific monitoring well.

2. If "U" is greater than three, the null hypothesis is accepted, and it cannot be concluded, with 90 percent or greater confidence, that the concentration for the individual contaminant has decreased with time at the specific monitoring well.

* Adapted from Mann, H. B. and Whitney, D.R., 1947, On a test of whether one of two random variables is stochastically larger than the other., Ann. Math. Statist., 18, pp. 52-54.

EXAMPLE 1: All data points are numerically unique

1. Individual Contaminant: TCE
   Individual Monitoring Well: MW-1

2. Monitoring quarters:

	$\hat{\theta}_1$ [Year 1]				$\hat{\theta}_2$ [Year 2]			
Sampling Round:	1	2	3	4	5	6	7	8
Sampling Result: (ppb) (concentration)	506a	1021a	612a	265a	543b	261b	77b	379b

3. 77b
   261b
   265a
   379b
   506a
   543b
   612a
   1021a

4. 265a=2, 506a=1, 612a=0, 1021a=0

5. 2+1+0+0=3, U=3

Conclusion: "U" is three, therefore the null hypothesis is rejected, and it is concluded, with 90 percent or greater confidence, that the first sampling set ($\hat{\theta}_1$) is greater than the second sampling set ($\hat{\theta}_2$), and therefore that the concentration for the specific contaminant in the specific monitoring well has decreased over the period of the ground water monitoring program.

EXAMPLE 2: two or more numerically identical data points

1. Individual Contaminant: TCE
   Individual Monitoring Well: MW-1

2. Monitoring quarters:

	$\hat{\theta}_1$ [Year 1]				$\hat{\theta}_2$ [Year 2]			
Sampling Round:	1	2	3	4	5	6	7	8
Sampling Result: (ppb) (concentration)	28a	NDa	61a	NDa	63b	NDb	77b	79b

3. a] NDb
   NDa
   NDa
   28a
   61a
   63b
   77b
   79b

   b] NDa
   NDa
   NDb
   28a
   61a
   63b
   77b
   79b

4. a] NDa=3, NDa=3, 28a=3, 61a=3
   b] NDa=4, NDa=4, 28a=3, 61a=3

5. a] 3+3+3+3=12 Ua=12==> U=13.0
   b] 4+4+3+3=14 Ub=14

Conclusion: "U" is thirteen, therefore we accept the null hypothesis, and we cannot conclude, with 90 percent or greater confidence, that the first sampling set ($\hat{\theta}_1$) is greater than the second sampling set ($\hat{\theta}_2$), and we cannot conclude that the concentration for that specific contaminant has decreased with time.